STAT 2593

Lecture 024 - Methods of Point Estimation

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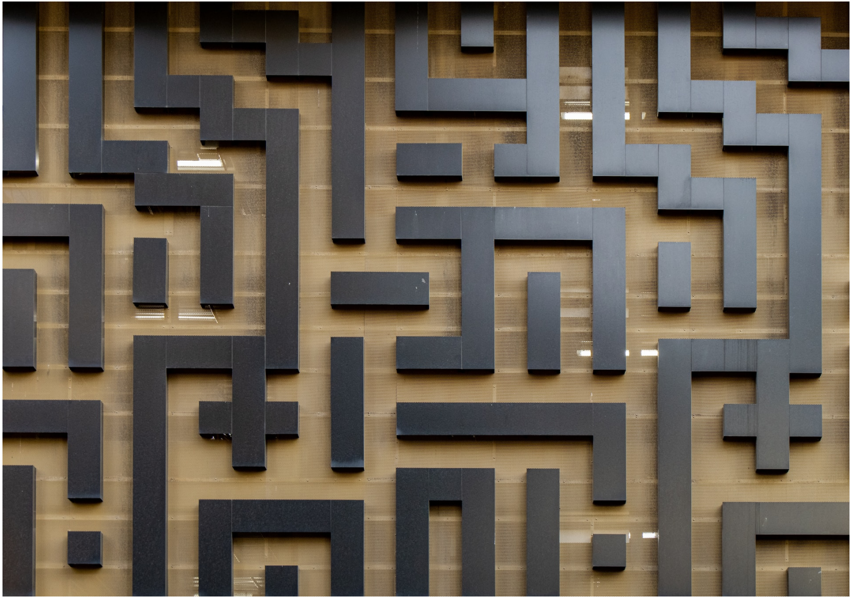
Methods of Point Estimation

Learning Objectives

1. Understand the method of moments estimation procedure.



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- Estimation techniques are designed to generate estimators for parameters of interest, generally.
  - **Maximum likelihood estimation** is the most prominent estimation technique.
  - **Method of moments** is an intuitive technique, which typically generates robust estimators.

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- Population moments will be functions of the unknown parameters, denoted θ.

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- With the estimator $\widehat{m}_k$, we can use a sample to compute estimates (i.e., values) for these.

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    3. We set $\widehat{m}_k = E[X^k]$, for $k = 1, \ldots, L$, and then solve for the components of Θ.
    4. The solution to the system of equations gives $\widehat{\Theta} = (\widehat{\theta}_1, \ldots, \widehat{\theta}_L)$.

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- The method of moments estimators assume that all of the population moments exist, this will not always be the case.
- The method of moments estimators will generally be **biased**.
- Method of moment estimators do not need to be **valid** parameter values.

Summary

- Estimation techniques are general processes which generate estimators.
- The method of moments estimators are derived by setting population moments equal to sample moments, and solving for the unknown parameters.
- Method of moments estimators are robust and intuitive, but generally biased, and may not exist (or be valid).